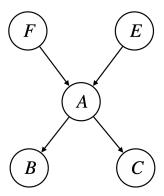
CSC 665: Artificial Intelligence

Homework 5

By turning in this assignment, I agree to abide by SFSU's academic integrity code and declare that all of my solutions are my own work.

1 Inference in Bayesian networks

Consider the alarm Bayesian network we studied in class.



Each of these random variables is binary-valued. F indicates whether your home is on fire, E indicates whether an earthquake is happening, A indicates whether your alarm system has gone off, B indicates whether your neighbor Bob has called to tell you that your alarm has sounded, and C indicates whether your neighbor Carol has called to tell you that your alarm has sounded.

The local conditional probability distributions for each of these random variables are as follows:

P	r(F)	=	1)
0.01			

$$P(E=1)$$

$$0.02$$

f	e	$ P(A=1 \mid F=f, E=e) $
1	1	0.95
1	0	0.94
0	1	0.29
0	0	0.01

a	$P(B=1 \mid A=a)$
1	0.90
0	0.05

a	$P(C=1 \mid A=a)$	
1	0.70	
0	0.01	

HW5

- a. (15 points) Perform exact inference by enumeration to compute $P(F = 1 \mid B = 1)$ and $P(F = 1 \mid C = 1)$. You may do this by hand or you may write a program, but please show your work in either case.
- b. (15 points) Write a program to perform rejection sampling to estimate $P(F=1 \mid B=1)$ and $P(F=1 \mid C=1)$. Perform separate runs of rejection sampling using N=10,100,1000,1000 samples. (Note that N refers to the total number of samples on a given run, many of which will be rejected.) Show the results of your sampling runs by producing a plot of the estimated value for each of the two probabilities as a function of N. Include the true value of each probability (computed in part (a)) in the plot as a horizontal line.
- c. (15 points) Write a program to perform likelihood weighting to estimate $P(F = 1 \mid B = 1)$ and $P(F = 1 \mid C = 1)$. Perform separate runs of likelihood weighting using N = 10,100,1000,10000 samples. Show the results of your sampling runs by producing a plot of the estimated value for each of the two probabilities as a function of N. Include the true value of each probability (computed in part (a)) in the plot as a horizontal line.
- d. (10 points extra credit) Make an insightful comment about some aspect of the results obtained above.

2 Gradient descent

Suppose we wish to use a linear model for a binary classification in which the label space is $\mathcal{Y} = \{-1, +1\}$. In this case, squared error is no longer an appropriate cost function (think about why). Instead, a common choice of pointwise cost is the *hinge loss*, defined by

$$c(\hat{y}, y) = \max\{0, 1 - \hat{y}y\},\$$

where $y \in \mathcal{Y}$ is the true label and $\hat{y} = h(x)$ is the prediction produced by our model.

- a. (10 points) Sketch a plot of $c(\hat{y}, +1)$ and $c(\hat{y}, -1)$ as a function of \hat{y} on the interval [-5, 5]. Based on these plots, briefly argue why the hinge loss is a reasonable cost function for binary classification.
- b. (20 points) Suppose that we have a single real-valued feature, i.e. $\mathcal{X} = \mathbb{R}$, and that our hypothesis class is the set of linear models of the form $h(x) = w_0 + w_1 x$. Compute

$$\frac{d}{dw_0}c(h(x),y)$$
 and $\frac{d}{dw_1}c(h(x),y)$,

where c is the hinge loss defined above.

c. (20 points) Consider the following training dataset:

i	x_i	y_i
1	-5	-1
2	1	+1
3	-2	-1
4	2	-1
5	4	+1
6	7	+1

Perform stochastic gradient by hand on this dataset to optimize the hinge loss with respect to the parameters of the linear model. Use the initialization $w_0 = w_1 = 0$. On the *i*th iteration of SGD, use the single training example (x_i, y_i) to update w_0 and w_1 with a learning rate of $\eta = 0.1$. Perform a total of six iterations of SGD (one for each example), showing the values of w_0 and w_1 after the update on each iteration.

- d. (15 points) Given a trained linear model h, we can make class predictions according to the sign of h. That is, given a new input x, we classify x as positive (+1) if $h(x) \ge 0$ and negative (-1) if h(x) < 0. Using the final weights from part (c) and this classification rule, what is the misclassification rate of h on the training dataset? What is the best achievable misclassification rate among the set of all possible linear models? If our learned h does not achieve this rate, explain why not.
- e. (10 points extra credit) Write a program to execute (non-stochastic) gradient descent for the linear model on the dataset above. Each iteration of gradient descent should use the entire dataset to update the weight parameters. As usual, let the cost function C for the entire dataset be the sum of the hinge losses c for each training example. Run your program to convergence of the parameter values. Compare the h learned here to the h learned in part (c).

Submission

Submission is done on Canvas. You should submit either two or three files: one containing your solutions to the written problems, one for the coding problem in question 1, and optionally one for the extra credit coding problem in question 2.

- Submit your written solutions in a single PDF file with your name at the top. Make sure to clearly indicate the number and letter of the problem corresponding to each solution. It is okay to hand-write your solutions and then scan them into a PDF, but only if your handwriting is legible.
- Submit your coding solutions to question 1 in a file named bn.py.
- (Optional) Submit your coding solution to question 2 (e) in a file named sgd.py.