Artificial Intelligence

CSC 665

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PGMs IV

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- **Search:** make decisions by looking ahead
- **Logic:** deduce new facts from existing facts
- **Constraints:** find a way to satisfy a given specification
- **Probability:** reason quantitatively about uncertainty
- **Learning:** make future predictions from past observations

Modeling

Factored representations

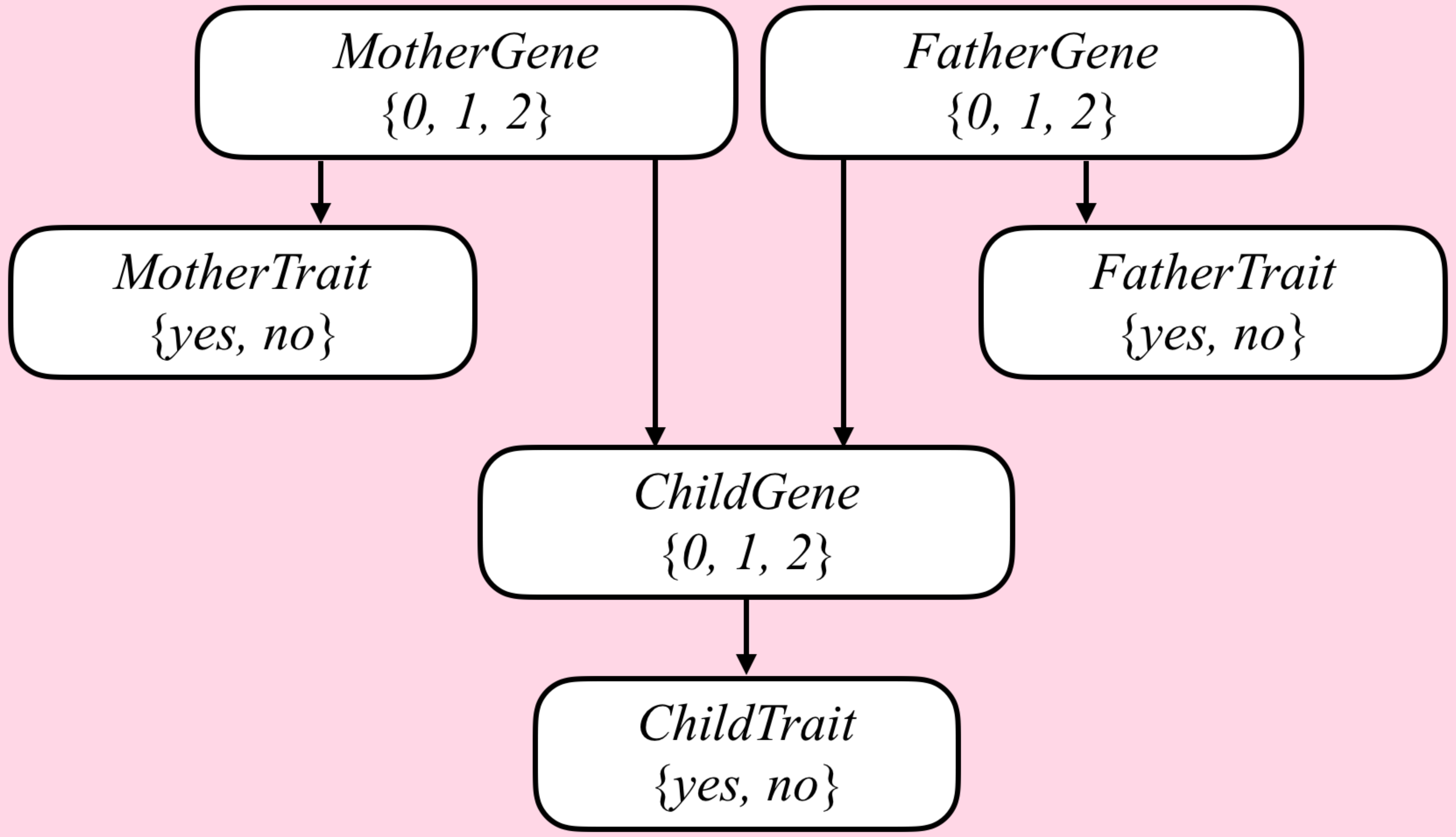
- Not just good for compactness!
- Factored representations make it easy to construct **complex models** from **simple parts**
- Saw this with **propositional** vs. **first-order** logic
- Propositional formulas are **somewhat decomposable**
- E.g., to understand $(p \wedge \neg q) \vee (q \wedge \neg p)$, examine each disjunct separately

Factored representations

- But propositional symbols are **atomic**, which **limits composability**
- E.g., "every CSC 665 student knows AI" is **awkward** to express in propositional logic: ArinKnowsAI $\land$ MarieKnowsAI $\land$ RoryKnowsAI $\land$ …
- Yet the statement in English is a **simple**, relating a set of students to a disciplinary field
- We needed to **extend our language** to first-order logic in order to get this additional level of compositionality

Factored representations

- Bayesian networks represent a joint distribution as a product of **simple conditional distributions**
- Imagine writing down the joint distribution for the 6 variables on the right one row at a time…
- But specifying the conditional distributions and then multiplying is (relatively) **straightforward**
- From **simple parts**, a **complex whole**



Inference

Types of inference

- Exact inference
  - Compute $P(X \mid E)$ exactly
  - Only tractable for small models with no continuous variables
- Approximate inference
  - Approximate $P(X \mid E)$
  - There's a chance the approximation is bad, and you have no way of knowing for sure

Exact inference by enumeration

- Divide set of all variables into
  - Query variables X
  - Evidence variables E
  - Other variables Y

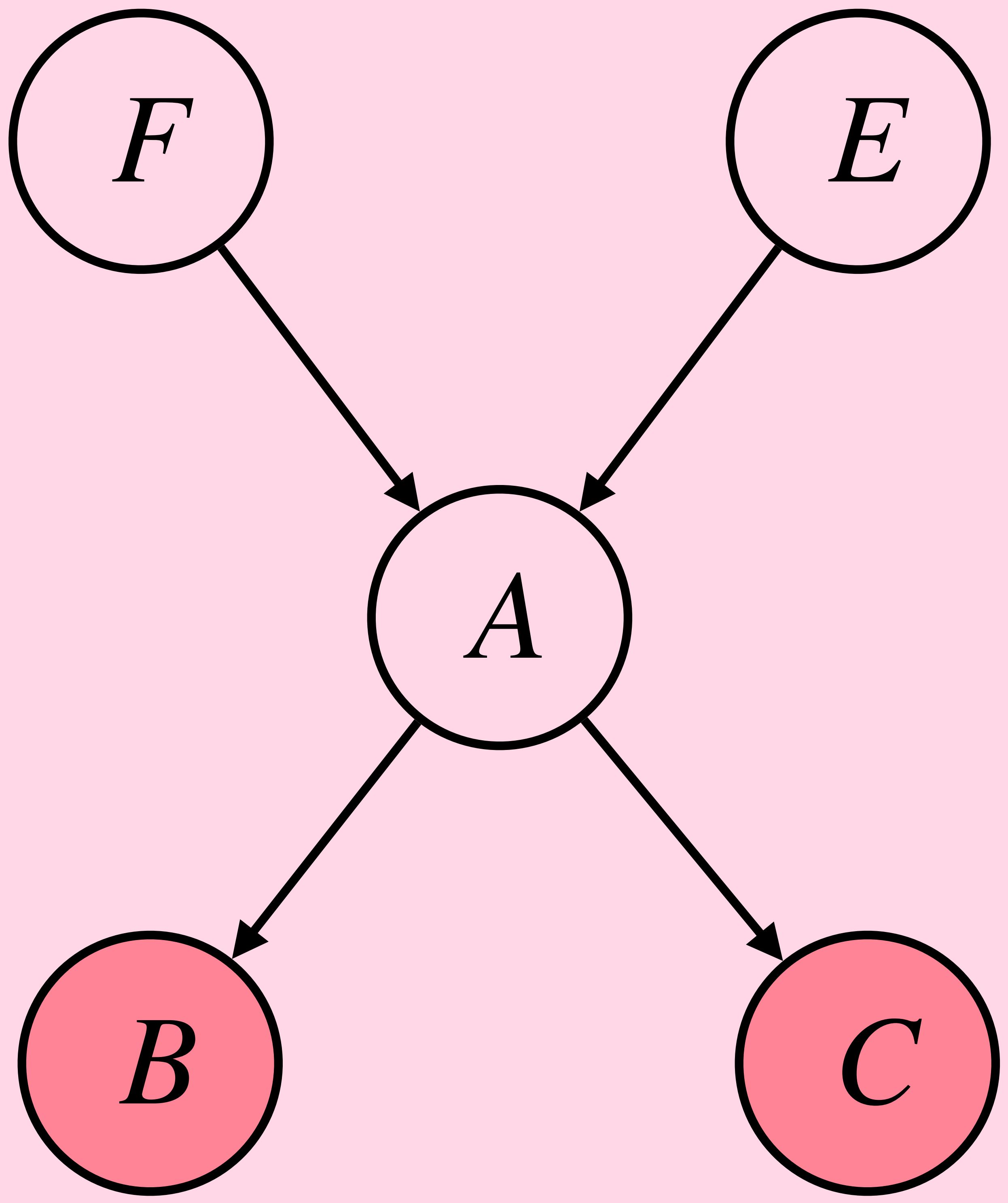
$$
\begin{aligned}
P(x \mid e) &= \frac{P(x, e)}{P(e)} \\
&= \alpha P(x, e) \\
&= \alpha \sum_{y} P(x, y, e)
\end{aligned}
$$

We know how to compute $P(x, y, e)$ from the Bayesian network

Exact inference by enumeration

- Query variables F
- Evidence variables B, C
- Other variables A, E

$$\begin{aligned} P(f \mid b, c) &= \alpha P(f, b, c) \\ &= \alpha \sum_a \sum_e P(f, e, a, b, c) \\ &= \alpha \sum_a \sum_e P(f)P(e)P(a \mid f, e)P(b \mid a)P(c \mid a) \end{aligned}$$



This requires us to sum 4 products of 5 terms each

In general, $O(2^n)$ products of $O(n)$ terms $\implies$ $O(n2^n)$ overall time complexity

Can get $O(2^n)$ with backtracking-like algorithm, but can't do any better

Exact inference by enumeration

function doInferenceByEnumeration(X, e)

- $Q(X) \leftarrow$ empty distribution over X
- **for** each value x of X:
  - $Q(x) \leftarrow \text{jointProb}(\{X, E, Y\}, \{X = x, E = e\})$ ← $P(X = x, E = e) = \sum_y P(X = x, E = e, Y = y)$
- **return** normalize($Q(X)$)

function jointProb(vars, assignments)

- **if** vars is empty: **return** 1.0
- $V \leftarrow$ first(vars)
- **if** V has an assignment v in assignments:
  - **return** $P(v \mid \text{parents}(V)) \cdot \text{jointProb}(\text{rest(vars)}, \text{assignments})$
- **else**: **return** $\sum_v P(v \mid \text{parents}(V)) \cdot \text{jointProb}(\text{rest(vars)}, \text{assignments} \cup \{V = v\})$

Sampling

- Suppose you have a coin $C \in \{h, t\}$
- You don't know if it's fair or biased, or what the bias parameter is
- How would you estimate $P(C = h)$?
- **Answer:** sample!
- Flip the coin N times. If there are n heads, estimate $P(C = h) \approx n/N$
- Is this a good estimator?
- Yes, in the sense that $n/N \to P(C = h)$ as $N \to \infty$
- The more samples we collect, the better the estimate

Estimating the joint distribution

- Can we sample from $P(X_1, \ldots, X_n)$ if we have its Bayesian network?
- Yes! As long as we can sample each conditional distribution (easy for discrete distributions)
- Algorithm:
  - assume $X_1, \ldots, X_n$ are in topological order
  - **for** $i = 1, \ldots, n$:
    - sample $x_i \sim P(X_i \mid \text{parents}(X_i))$, where the parents are assigned values from previous samples $x_1, \ldots, x_{i-1}$
  - **return** sample $(x_1, \ldots, x_n)$
- The relative frequency of a given assignment $(x_1, \ldots, x_n)$ approaches $P(x_1, \ldots, x_n)$ as more samples are generated