Artificial Intelligence

CSC 665

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PGMs III

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- **Search:** make decisions by looking ahead
- **Logic:** deduce new facts from existing facts
- **Constraints:** find a way to satisfy a given specification
- **Probability:** reason quantitatively about uncertainty
- **Learning:** make future predictions from past observations

Independence helps

- Independence reduces the amount of data needed to specify the joint distribution
- Factored representation $P(X, Y) = P(X)P(Y)$
- 36 → 12 numbers in the dice example
- In general: exponential → linear in the number of variables
- **Problem:** independence is rare in practice

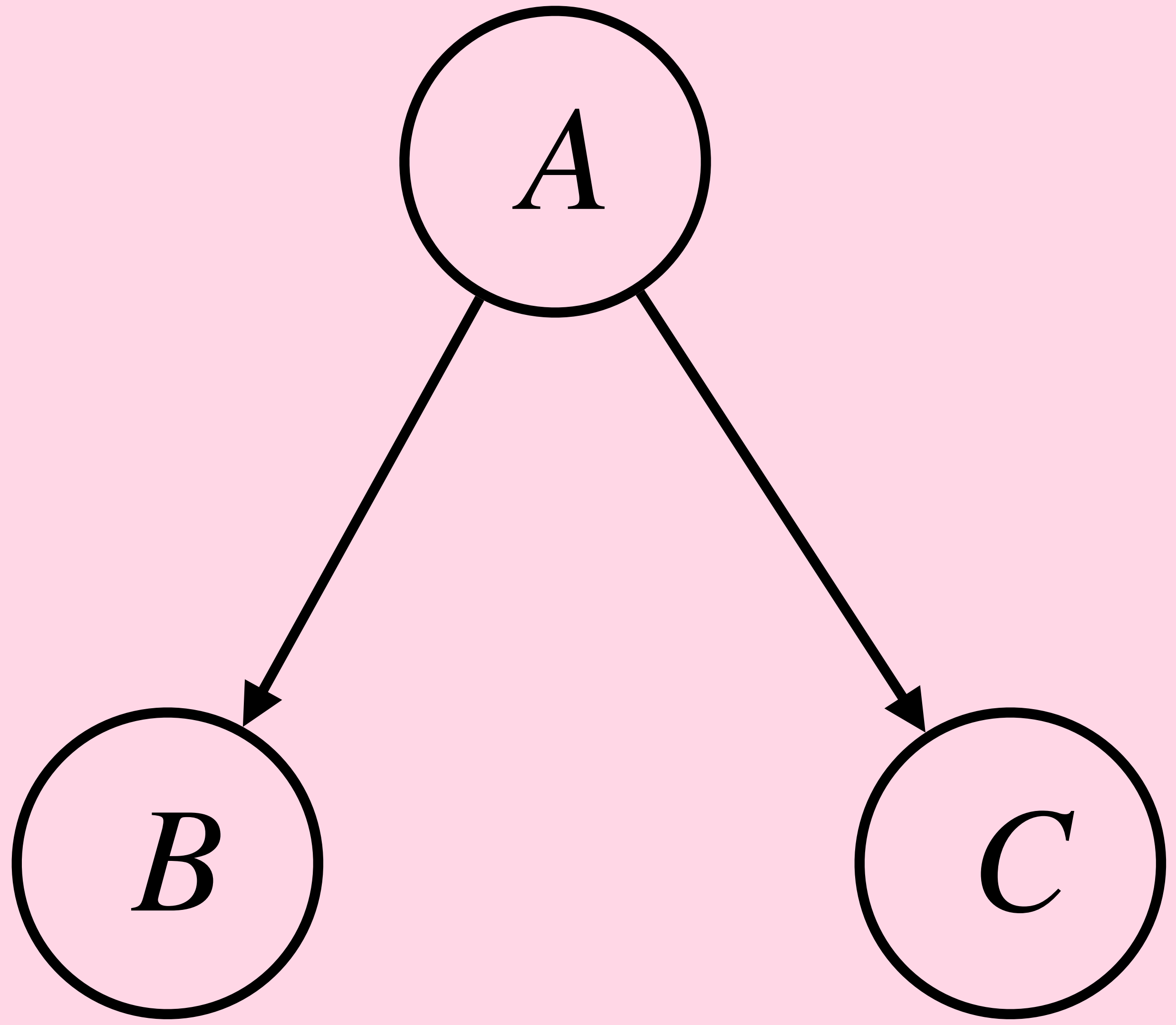
X	*Y*	*P(X,Y)*
1	1	1/36
1	2	1/36
…	…	…
6	6	1/36

X	*P(X)*
1	1/6
2	1/6
…	…
6	1/6

Y	*P(Y)*
1	1/6
2	1/6
…	…
6	1/6

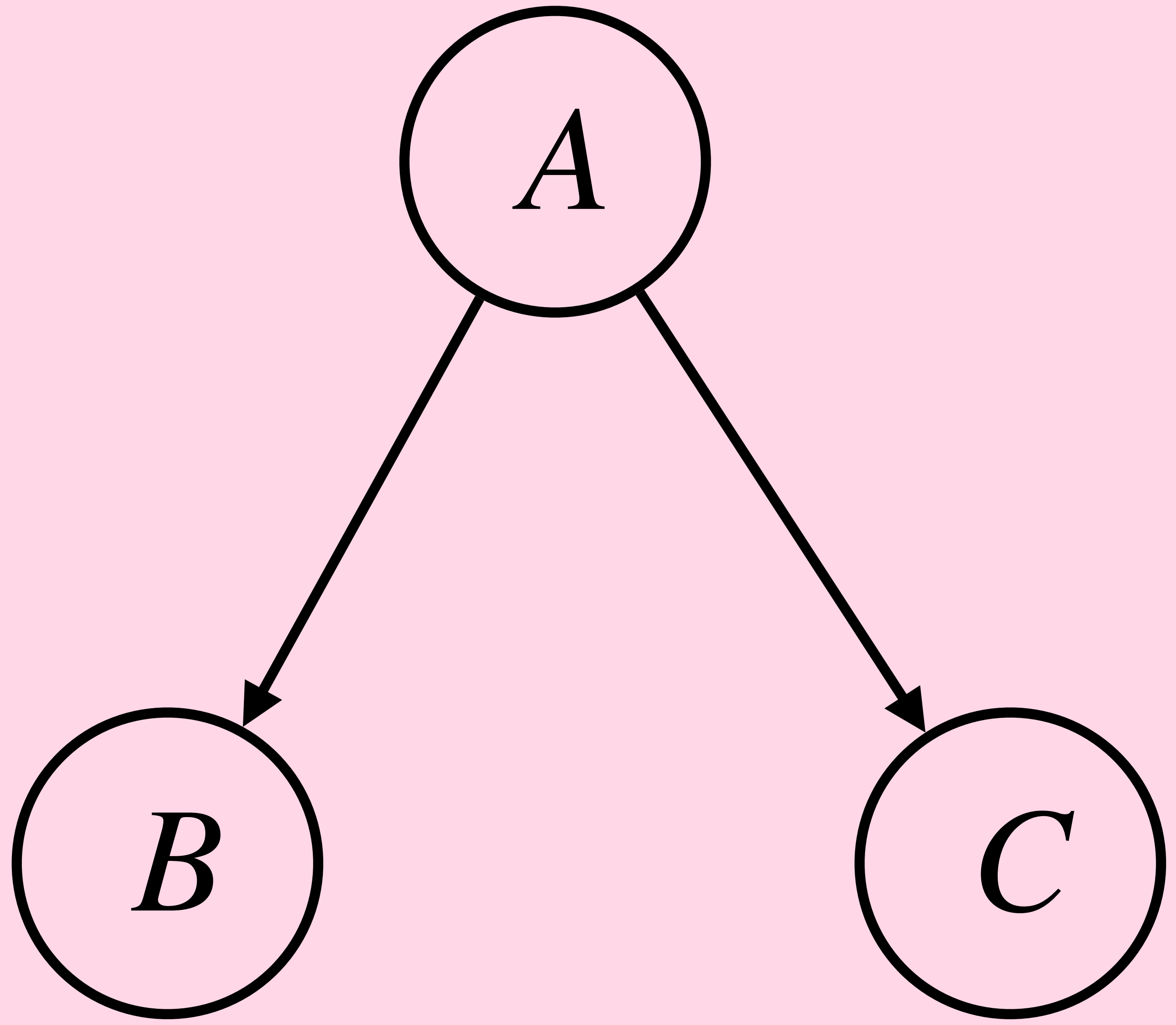
Example: alarm reporting

- Your house has an alarm system
- If the alarm goes off, your two neighbors Bob and Charlie will call you if they hear it
- Binary random variables A, B, C for alarm goes off, Bob calls, Charlie calls
- **Question:** are B and C independent?
- No! If Bob calls, then that means the alarm likely went off, so Charlie is likely to call too
- $P(C = 1 \mid B = 1) > P(C = 1)$



Example: alarm reporting

- B and C are not independent: $P(C \mid B) \neq P(C)$
- But they are *conditionally independent* given A
- If the alarm goes off, Charlie's ability to hear it is not affected by Bob
- $P(C \mid A, B) = P(C \mid A)$
- This allows for a factored joint distribution
- $P(B, C \mid A) = P(B \mid A)P(C \mid A)$

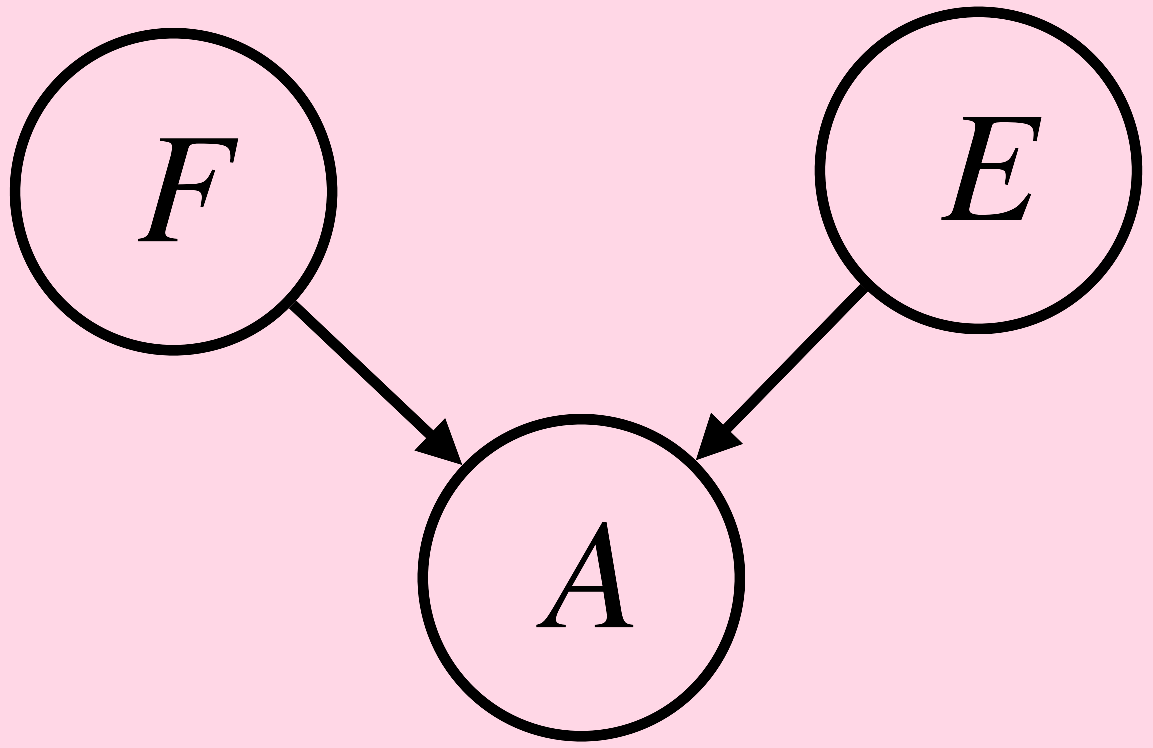


Conditional independence

- X and Y are conditionally independent given Z if $P(X, Y \mid Z) = P(X \mid Z)P(Y \mid Z)$
- Conditional independence is often easier to find in practice than unconditional independence
- **Important:** independence is different from conditional independence. Can have one without the other.

Example: alarm triggering

- Assume fires and earthquakes are independent rare events
- Either one will set off the alarm
- Joint distribution
  $P(F, E, A) = P(F)P(E)P(A \mid F, E)$
- Compute:
  - $P(E = 1 \mid A = 1)$
  - $P(E = 1 \mid A = 1, F = 1)$



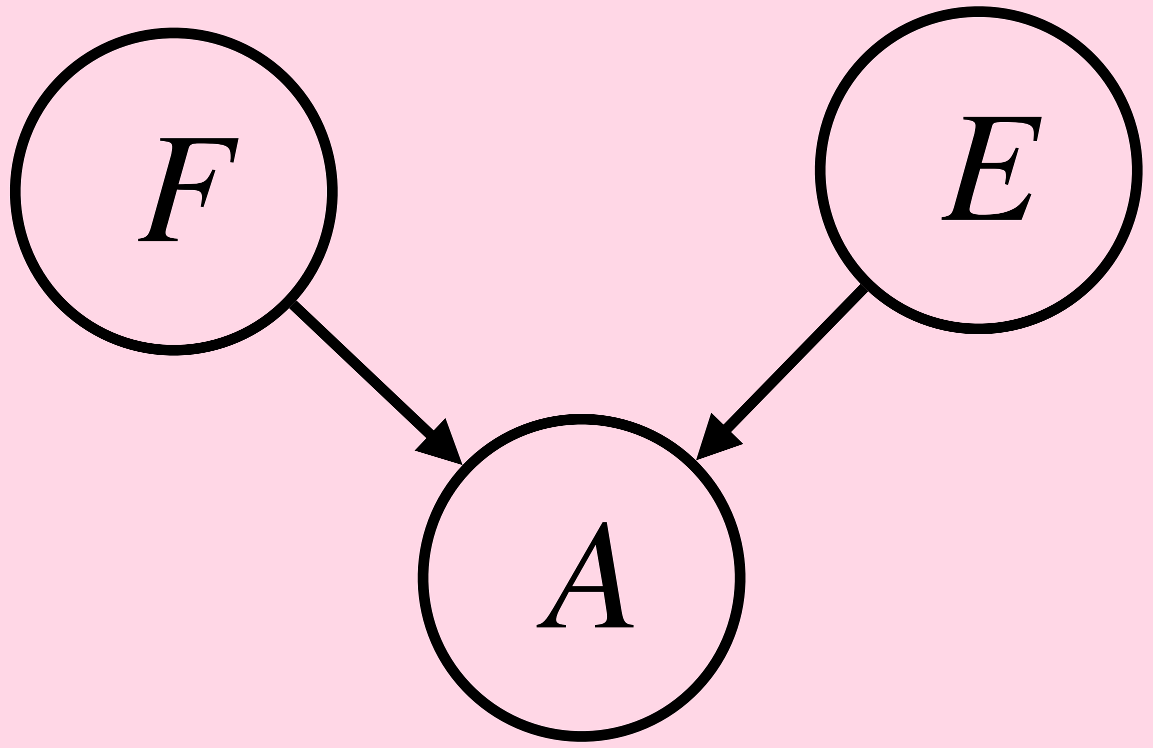
$P(F = 1)$
ϵ

$P(E = 1)$
ϵ

f	e	$P(A = 1 \mid F = f, E = e)$
0	0	0
0	1	1
1	0	1
1	1	1

Example: alarm triggering

- Assume fires and earthquakes are independent rare events
- Either one will set off the alarm
- Joint distribution
  $P(F, E, A) = P(F)P(E)P(A \mid F, E)$
- Compute:
  - $P(E = 1 \mid A = 1) = \frac{\epsilon(1-\epsilon) + \epsilon^2}{\epsilon(1-\epsilon) + \epsilon^2 + (1-\epsilon)\epsilon} = \frac{1}{2-\epsilon}$
  - $P(E = 1 \mid A = 1, F = 1) = \frac{\epsilon^2}{\epsilon^2 + (1-\epsilon)\epsilon} = \epsilon$



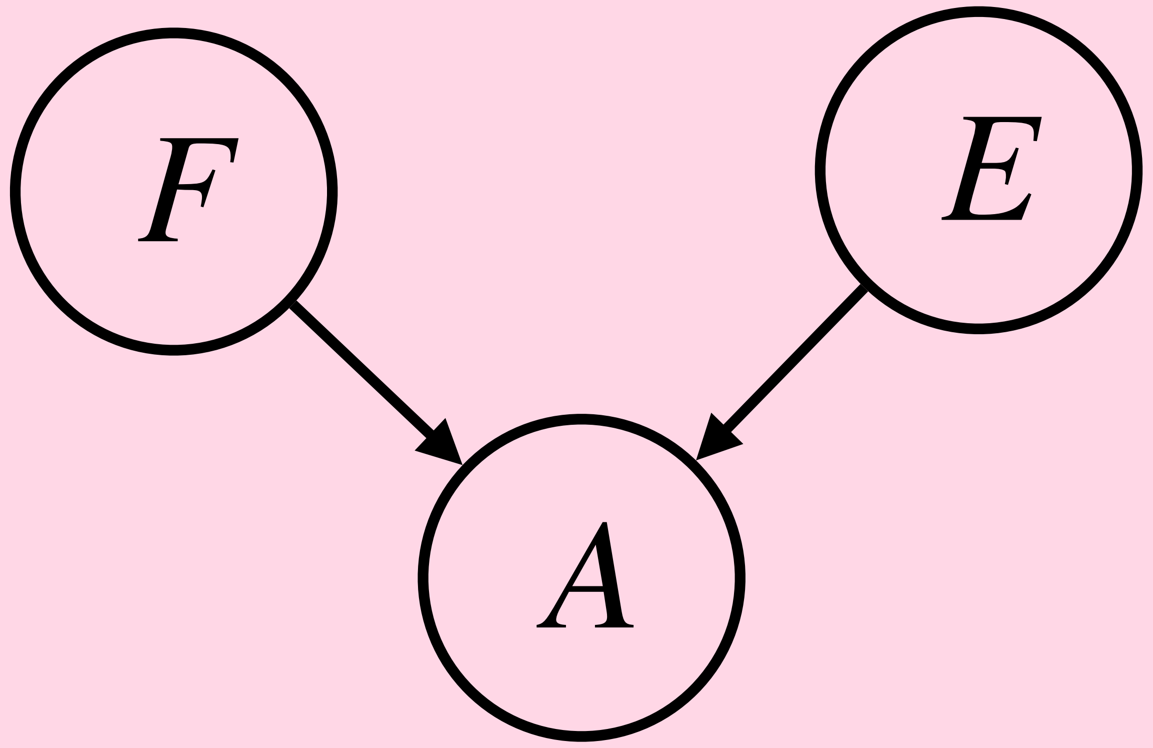
$P(F=1)$
ϵ

$P(E=1)$
ϵ

f	e	$P(A=1 \mid F=f, E=e)$
0	0	0
0	1	1
1	0	1
1	1	1

Example: alarm triggering

- $P(E = 1 \mid A = 1, F = 1) < P(E = 1 \mid A = 1)$
- **Conclusion:** If your alarm goes off, knowing there was a fire decreases the chance that there was an earthquake
- The fire has "explained away" the alarm
- Not a causal statement: fires do not protect against earthquakes!
- F and E are independent (unconditionally)
- But F and E are conditionally *dependent* given A!



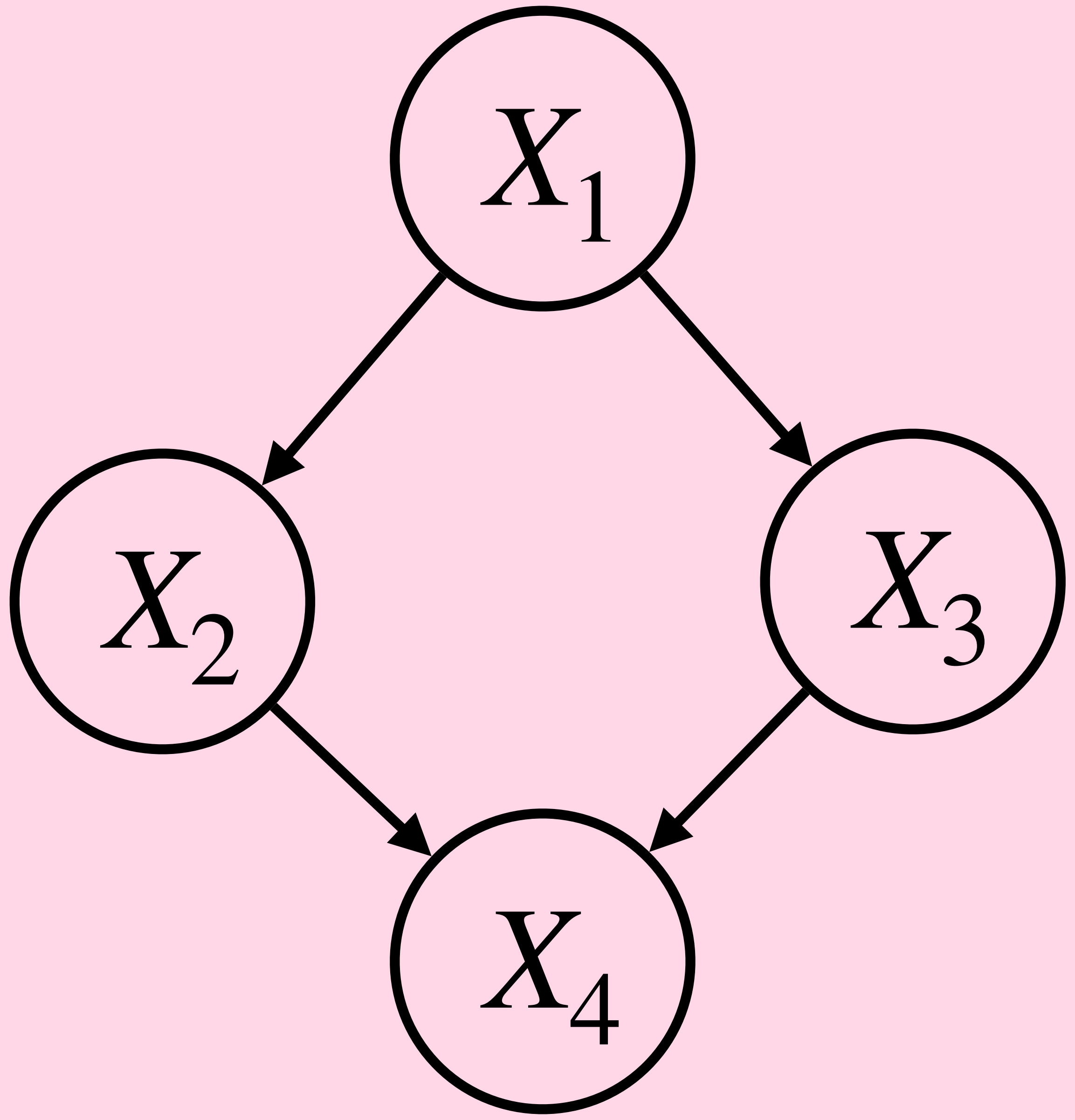
P(F = 1)
ϵ

P(E = 1)
ϵ

f	**e**	**P(A = 1 | F = f, E = e)**
0	0	0
0	1	1
1	0	1
1	1	1

Bayesian networks

- Let $X = (X_1, \ldots, X_n)$ be random variables
- A Bayesian network is a directed acyclic graph (DAG) where each node is a random variable
- The Bayesian network specifies a joint distribution over X as a product of local conditional distributions, one for each node
- $P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid \text{parents}(X_i))$



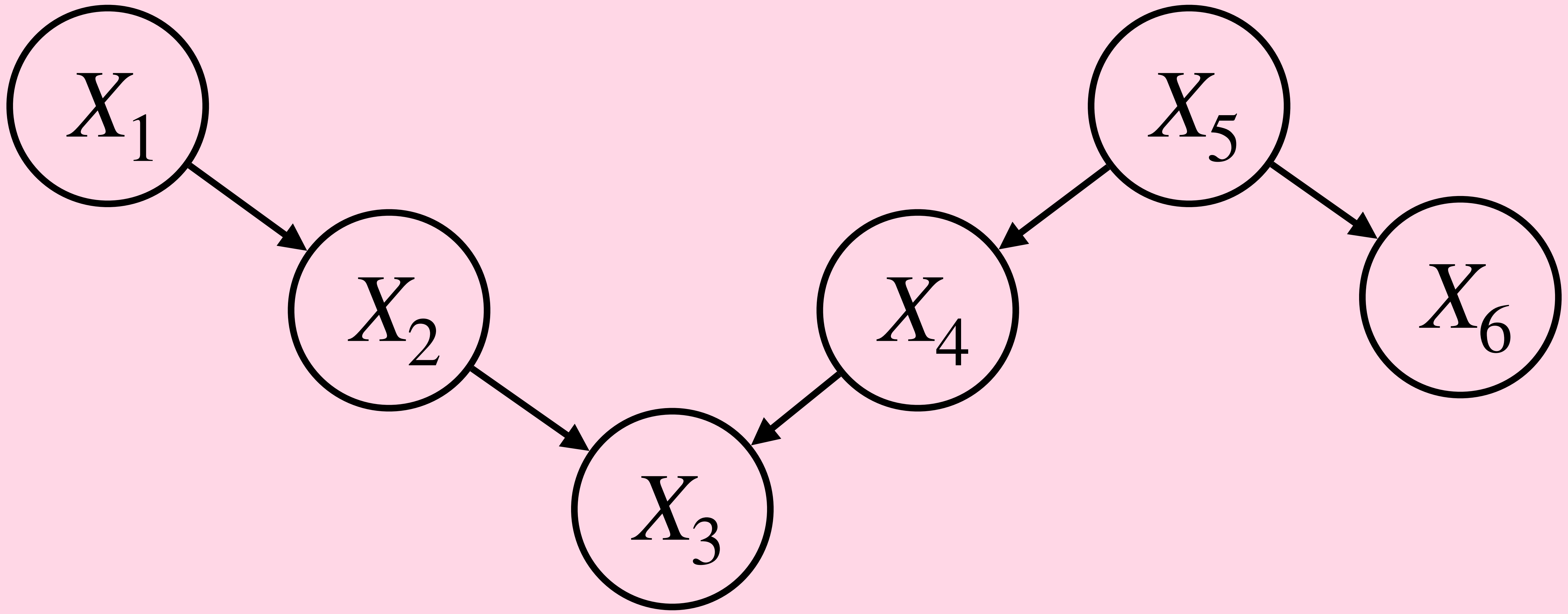
$$P(X_1, X_2, X_3, X_4) = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1)P(X_4 \mid X_2, X_3)$$

d-separation

- A Bayesian network lets us read off the conditional independence relationships between any pair of variables
- d-separation in the graph $\iff$ conditional independence in the joint distribution

d-separation rules

- **Rule 1:** unconditional d-separation
- X and Y are d-separated if there is no unblocked path between them
- A path is blocked if it contains two arrows colliding head-to-head in a v-structure

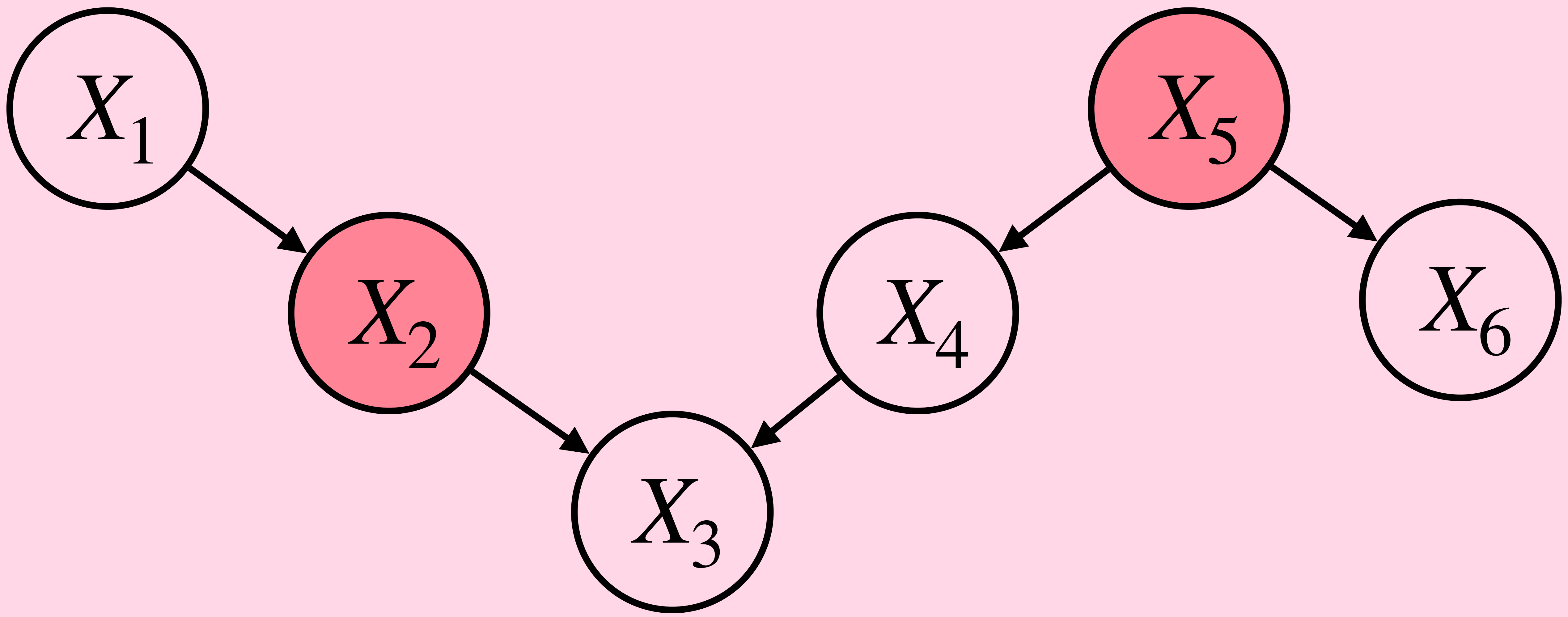


d-separated: X_2 and X_4, X_1 and X_6

not d-separated: X_1 and X_3, X_4 and X_5, X_3 and X_6

d-separation rules

- **Rule 2:** blocking by conditioning
- An unblocked path becomes blocked if one of the nodes in the path is observed (shaded)

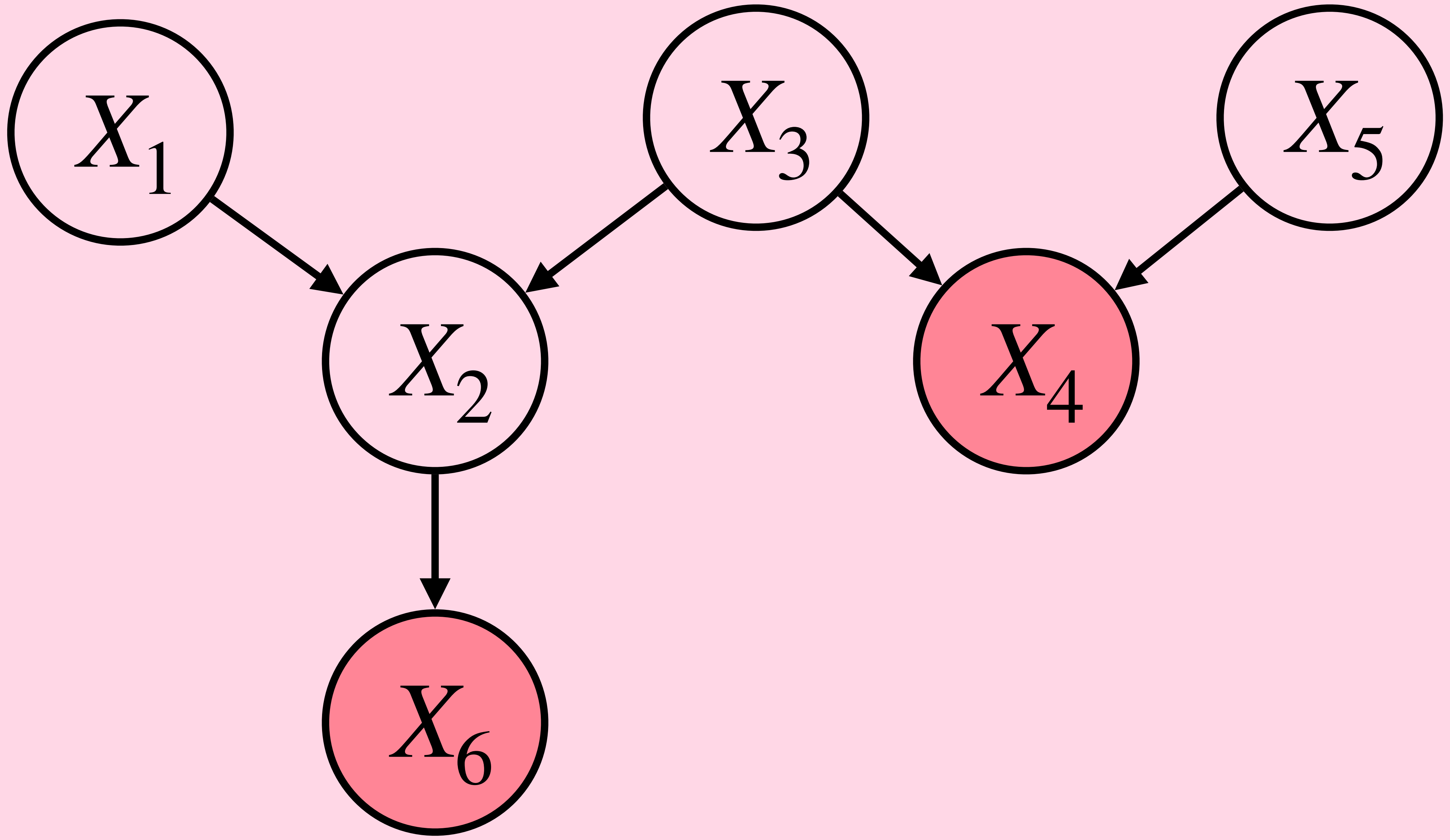


d-separated: X_1 and X_3, X_3 and X_6, X_1 and X_6

not d-separated: X_3 and X_4

d-separation rules

- **Rule 3:** activated v-structures
- If the node at the center of a v-structure or one of its descendants is observed, then the v-structure is no longer blocking

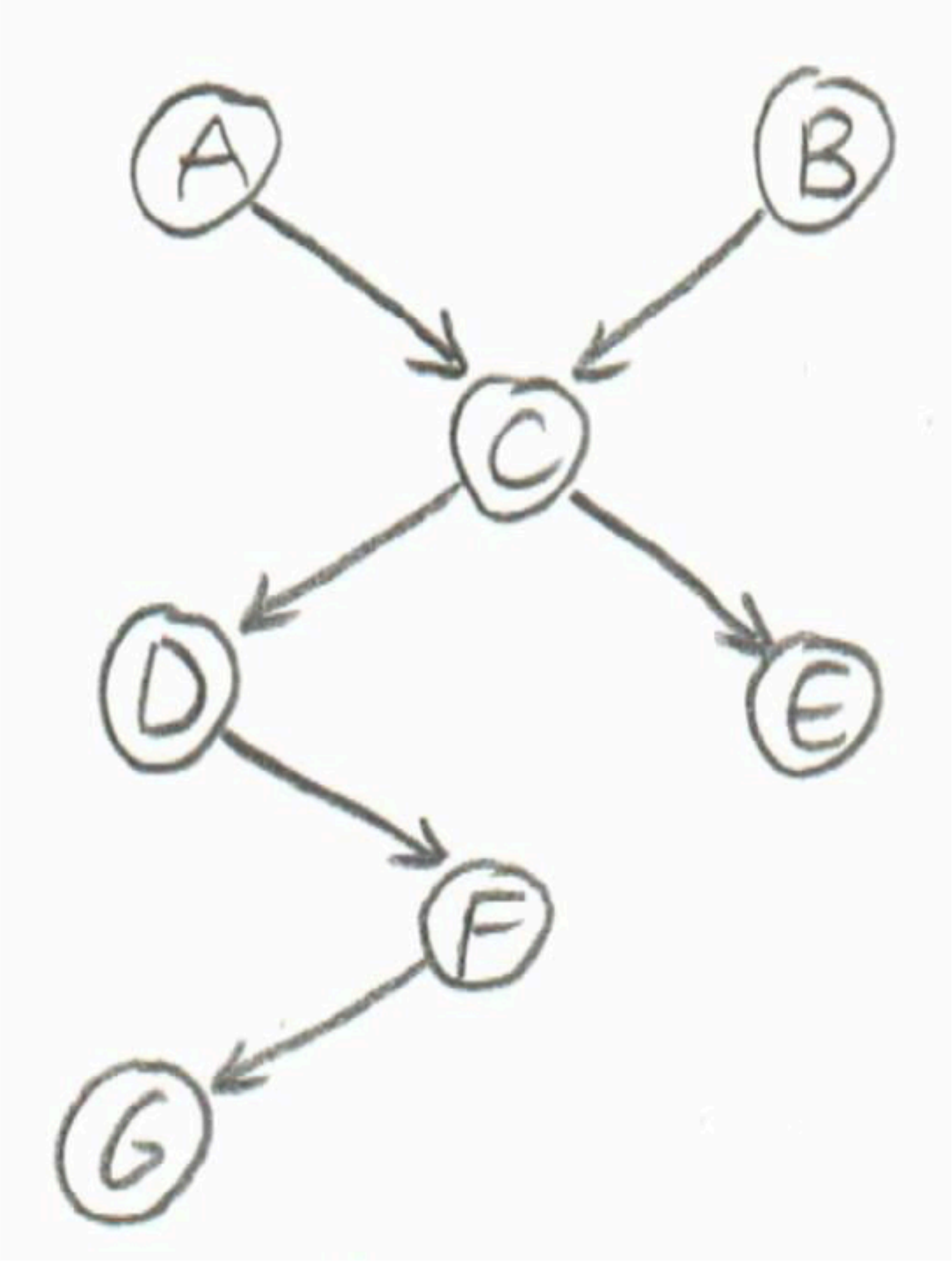


d-separated: nothing

not d-separated: everything

d-separation examples

- A and B, given D and F
- A and B
- A and B, given C
- D and E, given C
- D and E
- D and E, given A and B



d-separation examples

- A and B, given D and F
- **A and B**
- A and B, given C
- **D and E, given C**
- D and E
- D and E, given A and B

(**bold** indicates d-separation)